From $\sigma_{eq}$ we see that

\[ x_{eq} = (e^{\beta P} + e^{-\beta P})^{-1} (e^{-\beta P} - e^{\beta P}) \sin \frac{\Theta}{2} \cos \frac{\Phi}{2} 2 \cos \Phi \]
\[ y_{eq} = (e^{\beta P} + e^{-\beta P})^{-1} (e^{-\beta P} - e^{\beta P}) \sin \frac{\Theta}{2} \cos \frac{\Phi}{2} 2 \sin \Phi \]
\[ z_{eq} = (e^{\beta P} + e^{-\beta P})^{-1} \left[ e^{-\beta P} \left( \cos \frac{\Theta}{2} - \sin \frac{\Theta}{2} \right) - e^{\beta P} \left( \cos \frac{\Theta}{2} + \sin \frac{\Theta}{2} \right) \right] \]

\[ x_{eq} = -\tanh \beta P \sin \Theta \cos \Phi \]
\[ y_{eq} = -\tanh \beta P \sin \Theta \sin \Phi \]
\[ z_{eq} = -\tanh \beta P \cos \Theta \]

In the weak energy transfer limit

\[ \cos \Theta = \frac{\Delta}{\Delta (1 + 15/2 \Delta^2)} \approx 1 - \frac{15}{2 \Delta^2} \]

\[ \sin \Theta \approx 15 / \Delta \]

so our expressions for $x_{eq}$, $y_{eq}$, and $z_{eq}$ agree with RS Eq. (3.32) in this limit.
In order to Laplace transform the equations of motion for \( x(t), y(t), \) and \( z(t) \) (pages 25, 28, and 29) we use the results that

\[
\int_0^\infty \! dt \, e^{-pt} \frac{df(t)}{dt} = \int_0^\infty \! dt \left\{ \frac{d}{dt} (e^{-pt} f(t)) + p e^{-pt} \hat{f}(t) \right\}
\]

\[
= -f(0) + p \int_0^\infty \! dt \, e^{-pt} \hat{f}(t)
\]

\[
= p\hat{f}(p) - f(0)
\]

where we denote the Laplace transform with a hat:

\[
\hat{f}(p) = \int_0^\infty \! dt \, e^{-pt} f(t).
\]

We also make use of the convolution theorem:

\[
\int_0^\infty \! dt \, e^{-pt} \int_0^t \! dr \, f(r) g(t-r) = \int_0^\infty \! dr \, e^{-pr} f(r) \int_0^\infty \! dt \, e^{-p(t-r)} g(t-r)
\]

\[
= \hat{f}(p) \hat{g}(p)
\]
With initial conditions \( \chi(0) = y(0) = 0 \) and \( z(0) = 1 \), the transformed equations of motion become

\[
-p\hat{z}(p) - 1 = z\hat{y} + z\hat{x}(p) - \frac{B(p)}{p} - A(p)\hat{z}(p) - C(p)\hat{x}(p) - D(p)\hat{y}(p).
\]

This looks OK; seems to agree w/ corrected RS (3.33).

\[
p\hat{x}(p) = -2\hat{y}(p) - 2z\hat{\hat{z}}(p) - \frac{\hat{\hat{\hat{z}}}(p)}{p} - \beta(p)\hat{z}(p) - \delta(p)\hat{x}(p) - \hat{\delta}(p)\hat{y}(p).
\]

Seems OK by comparison w/ corrected RS (3.33).

\[
p\hat{y}(p) = z\hat{x}(p) - z\hat{\hat{\hat{z}}}(p) - \frac{\hat{\hat{\hat{x}}}(p)}{p} - \beta'(p)\hat{z}(p) - \delta'(p)\hat{y}(p) - \hat{\delta}'(p)\hat{x}(p).
\]

Looks OK.

Now the Laplace transform can be rewritten as

\[
\hat{g}(p) = \int_0^\infty dt e^{-pt}(g(t) - g(\infty) + g(\infty))
\]

\[
= \frac{g(\infty)}{p} + \int_0^\infty dt e^{-pt}(g(t) - g(\infty)).
\]

Since the integral in this expression remains finite at \( p = 0 \), we see that \( \hat{g}(p) \) has a simple pole at the origin with a residue equal to the long-time value of \( g(t \to \infty) \).
The Laplace-transformed eqns. of motion can be written in matrix form:

\[
\begin{bmatrix}
Z \Delta - \hat{\delta}'(p) & -p - \hat{\delta}'(p) & -2 \bar{J}_R - \hat{\beta}'(p) \\
-p - \hat{\delta}(p) & -Z \Delta - \hat{\delta}(p) & -2 \bar{J}_I - \hat{\beta}(p) \\
2 \bar{J}_I - \hat{\gamma}(p) & 2 \bar{J}_R - \hat{\delta}(p) & -p - \hat{\lambda}(p)
\end{bmatrix}
\begin{bmatrix}
\hat{x}(p) \\
\hat{y}(p) \\
\hat{z}(p)
\end{bmatrix}
=
\begin{bmatrix}
\frac{2'(p)}{p} \\
\frac{2'(p)}{p} \\
\frac{\hat{B}'(p)}{p} - 1
\end{bmatrix}
\]

All the time-dependent coefficients \(A(\tau)\) through \(D(\tau)\), \(\delta(\tau)\) through \(\delta'(\tau)\), and \(\delta'(\tau)\) through \(\delta''(\tau)\) vanish as \(\tau \to \infty\). So their Laplace transforms all remain finite as \(p \to 0\), and we can write

\[
\begin{bmatrix}
Z \Delta - \hat{\delta}'(0) & -\hat{\delta}'(0) & -2 \bar{J}_R - \hat{\beta}'(0) \\
-p - \hat{\delta}(0) & -Z \Delta - \hat{\delta}(0) & -2 \bar{J}_I - \hat{\beta}(0) \\
2 \bar{J}_I - \hat{\gamma}(0) & 2 \bar{J}_R - \hat{\delta}(0) & -\hat{\lambda}(0)
\end{bmatrix}
\begin{bmatrix}
\chi(\infty) \\
\gamma(\infty) \\
\hat{z}(\infty)
\end{bmatrix}
=
\begin{bmatrix}
\frac{2'(0)}{p} \\
\frac{2'(0)}{p} \\
\frac{\hat{B}'(0)}{p}
\end{bmatrix}
\]

\[\text{See pp. 10-11 for an explicit calculation of one coefficient (}\hat{B}(\tau)\), see pp. 10-11.\]
Do $x_{eq}$, $y_{eq}$, and $z_{eq}$ (from p. 33) work as steady-state solutions?

Through second order in $|\Gamma|/\Delta$ we have

$$
\begin{bmatrix}
2\Delta & 0 & -2\bar{\Gamma}_E \\
0 & -2\Delta & -2\bar{\Gamma}_I \\
2\bar{\Gamma}_I & 2\bar{\Gamma}_E & -\hat{\Lambda}(0)
\end{bmatrix}
\begin{bmatrix}
-x_{eq} \\
y_{eq} \\
z_{eq}
\end{bmatrix}
= 
\begin{bmatrix}
-tanh\beta F \cdot \frac{|\Gamma|}{\Delta} \\
tanh\beta F \cdot \frac{|\Gamma|}{\Delta} \\
-tanh\beta F
\end{bmatrix}
\begin{bmatrix}
\bar{\Gamma}_E \\
\bar{\Gamma}_I \\
\bar{\Gamma}_E
\end{bmatrix} 
$$

see p. 10

$$
\begin{bmatrix}
0 \\
0 \\
\hat{\Lambda}(0) \ tanh\beta F
\end{bmatrix} =
\begin{bmatrix}
0 \\
0 \\
\hat{B}(0)
\end{bmatrix}
$$

as required

see p. 12

So where does Förster theory come from?

From p. 25, the rate of decay of the population difference is

$$
\dot{Z}(t) \approx -\int_0^t A(\tau) Z(t-\tau) d\tau \approx -\int_0^\infty d\tau A(\tau) Z(t-\tau) = -\hat{\Lambda}(0) Z(t)
$$